

Theory of the spin Hall effect, and its inverse, in a ferromagnetic metal near the Curie temperature

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We give a theory of the inverse spin Hall effect (ISHE) in ferromagnetic metals based on skew scattering via collective spin fluctuations. This extends Kondo's theory of the anomalous Hall effect (AHE) to include short-range spin-spin correlations. We find a relation between the ISHE and the four-spin correlations near the Curie temperature T_C . Such four-spin correlations do not contribute to the AHE, which relates to the three-spin correlations. Thus our theory shows an essential difference between the AHE and ISHE, providing an essential complement to Kondo's classic theory of the AHE in metals. We note the relation to skew-scattering mechanisms based on impurity scattering. Our theory can be compared to recent experimental results by Wei *et al.* [Nat. Commun. **3**, 1058 (2012)] for the ISHE in ferromagnetic alloys.

The spin Hall effect (SHE), which converts charge current into spin current, along with the inverse effect (ISHE) which re-converts the currents, is one of the key phenomena for the further development of spintronic devices [1, 2]. A difficulty in exploiting the effect is that it depends on a spin-orbit interaction which, as a relativistic effect, is intrinsically weak except in heavy elements. Recently the doping of gold with low densities of iron impurities was proposed as a mechanism to enhance the SHE through spin fluctuations of the individual iron moments [3, 4]. This raises the question as to whether the effect might also be sensitive to *collective* magnetic fluctuations, in particular, close to a magnetic phase transition, where the fluctuations are strong, in analogy with the anomalous Hall effect (AHE) in a pure ferromagnetic metal such as Fe [5] and Ni [6], where a peak of the Hall resistivity appears below the Curie temperature T_C . Wei *et al.* [7] observed a feature in the ISHE, scaling with T_C for different samples of a metallic alloy, which suggested that this is in fact so, but the form is quite different from that of the AHE.

The observations of the AHE near the Curie temperature were explained by Kondo in his calculations [8] of an s - d (or s - f) lattice where, apart from the standard Coulomb interactions between the orbitals, he included on-site spin-orbit interactions. This gives rise to skew scattering in the conducting s band. We will show here that while both the AHE and SHE are generated by skew scattering that is linear in the spin-orbit term, they have *qualitatively* different behaviors near T_C as they depend on correlations of distinct order. In Kondo's formulation Coulomb interactions are included by keeping s - and p -scattering channels for the conducting electrons around each localized orbital. The angular momentum can thereby be transferred from the local moment to the conduction electrons. As we are interested in the effects of skew scattering, we will write only terms depending on the vector product $\boldsymbol{\kappa}' \times \boldsymbol{\kappa}$, where $\boldsymbol{\kappa}$ (respectively $\boldsymbol{\kappa}'$) is the unit vector in the direction of \mathbf{k} (respectively \mathbf{k}'),

the momentum of an electron in the s band. The s - d exchange interaction is then [8] (omitting terms depending on quadrupole moments that do not contribute here)

$$H = - \sum_{n, \mathbf{k}, \mathbf{k}', \mu, \mu'} N^{-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{R}_n} a_{\mathbf{k}\mu}^* a_{\mathbf{k}'\mu'} \left[(3i/4) F_2 \mathbf{L}_n \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa}) + 2(\mathbf{S}_n \cdot \mathbf{s}_c) \{ J(\mathbf{k}, \mathbf{k}') + (3i/2) c_2 F_2 \mathbf{L}_n \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa}) \} \right].$$

\mathbf{L}_n and \mathbf{S}_n are the total orbital and spin angular momenta of the localized d orbital on atomic site n at position \mathbf{R}_n , of a total number of N magnetic atoms in the crystal. \mathbf{s}_c is the spin operator for the conduction electrons. $J(\mathbf{k}, \mathbf{k}') = F_0 + 2F_1(\boldsymbol{\kappa} \cdot \boldsymbol{\kappa}')$ with $F_{0,1,2}$ exchange terms generated by Coulomb interactions between the localized d electrons and the different channels of scattering with propagating s orbitals. The coefficients c_2 appear in the summation of individual orbital and spin angular momenta of each electron within the localized orbitals to give the total momentum [8]. Assuming an orbital singlet but adding a small spin-orbit $H_{LS} = \lambda \sum_n \mathbf{L}_n \cdot \mathbf{S}_n$, it generates operators \mathbf{L}_n on each site and gives a corrected Hamiltonian with the coefficient Λ_1 linear in the spin-orbit coupling λ and determined by the local crystal field levels [8],

$$H = - \sum_{n, \mathbf{k}, \mathbf{k}', \mu, \mu'} N^{-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{R}_n} a_{\mathbf{k}\mu}^* a_{\mathbf{k}'\mu'} \left[2(\mathbf{S}_n \cdot \mathbf{s}_c) J(\mathbf{k}, \mathbf{k}') + i\Lambda_1 F_2 \{ \mathbf{S}_n \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa}) + 2c_2 (\mathbf{S}_n \cdot \mathbf{s}_c) (\mathbf{S}_n \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa})) \} + 2c_2 (\mathbf{S}_n \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa})) (\mathbf{S}_n \cdot \mathbf{s}_c) - 4c_2/3 (\mathbf{S}_n \cdot \mathbf{S}_n) (\mathbf{s}_c \cdot (\boldsymbol{\kappa}' \times \boldsymbol{\kappa})) \} \right]. \quad (1)$$

Considering only elastic scattering, the matrix element

of the above Hamiltonian is [8],

$$H_{\mathbf{k}\pm, \mathbf{k}'\pm} = - \sum_n N^{-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{R}_n} \left[\pm J(\mathbf{k}, \mathbf{k}') (M_n - \langle M_n \rangle) \right. \\ \left. + i\Lambda_1 F_2(\boldsymbol{\kappa}' \times \boldsymbol{\kappa})_{\boldsymbol{\zeta}} \{ \pm 2c_2(M_n^2 - \langle M_n^2 \rangle) + (M_n - \langle M_n \rangle) \} \right]. \quad (2)$$

M_n is the magnetization of the localized electron of the n -th ion, and $\boldsymbol{\zeta}$ is the magnetization direction below T_C or the direction of polarization of the spin current above, a direction which will be determined by the spin injector in the devices used. Subtractions of the lattice-averaged powers of the magnetization at each site, $\langle M_n \rangle$ and $\langle M_n^2 \rangle$, appear because any modification of the crystalline potential simply renormalizes the Bloch functions of the conduction electrons and does not contribute to scattering. There are two parts in Eq. (2): One is the $J(\mathbf{k}, \mathbf{k}')$ term, coming from the s - d scattering, and the other is the Λ_1 term, which is linear in spin-orbit coupling λ . \pm are spin states of the conduction electron. $(\boldsymbol{\kappa}' \times \boldsymbol{\kappa})_{\boldsymbol{\zeta}}$ comes from the approximation of including elastic scattering only.

In order to find a net current transverse to the voltage, the transition probabilities must be calculated to at least the second Born approximation, *i.e.*, the transition probabilities are to third power in the Hamiltonian. There is a triple summation over sites which are reduced, with Kondo's assumption of uncorrelated fluctuations, to a single summation. Now we generalize to include correlations but for simplicity we include those with only two-site indices, but all powers of spin. The transition probability from $\mathbf{k}'\pm$ to $\mathbf{k}\pm$ is given by

$$W(\mathbf{k}'\pm, \mathbf{k}\pm) = \delta(E_k - E_{k'}) \{ U_1 + U_3 + V^\pm(\mathbf{k}', \mathbf{k}) \},$$

where the s - d scattering contribution of U_1 is given by

$$U_1 = (2\pi/\hbar N) |J(\mathbf{k}, \mathbf{k}')|^2 p_1, \\ p_1 = \sum_{n=0}^{N-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot (\mathbf{R}_n - \mathbf{R}_0)} \langle (M_n - \langle M_n \rangle) (M_0 - \langle M_0 \rangle) \rangle.$$

A contribution $U_3 = (2\pi/\hbar N) P(\mathbf{k}, \mathbf{k}')$ is included to represent non-magnetic contributions, static impurity scattering, or phonons, where $P(\mathbf{k}, \mathbf{k}')$ is assumed to depend only on the angle between \mathbf{k} and \mathbf{k}' , and not to show significant temperature dependence close to the magnetic phase transition. The spin-orbit contribution of $V^\pm(\mathbf{k}', \mathbf{k})$ is obtained as

$$V^\pm(\mathbf{k}', \mathbf{k}) = (2\pi/\hbar) (\Lambda_1 F_2 V / 3\pi N^2) (2m^3)^{1/2} \hbar^{-3} E_k^{1/2} \\ \times (\boldsymbol{\kappa}' \times \boldsymbol{\kappa})_{\boldsymbol{\zeta}} \left[(r_1 \pm r_{2a}) \{ 3F_0^2 + 4F_1^2(\boldsymbol{\kappa} \cdot \boldsymbol{\kappa}') \} + (r_1 \pm r_{2b}) \right. \\ \left. \times \{ -4F_0 F_1 - 8F_1^2(\boldsymbol{\kappa} \cdot \boldsymbol{\kappa}') \} \right], \quad (3)$$

where r_1 , r_{2a} , and r_{2b} depend on $\mathbf{k}' - \mathbf{k}$ and temperature,

$$r_1 = \sum_{n=0}^{N-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot (\mathbf{R}_n - \mathbf{R}_0)} \langle (M_n - \langle M_n \rangle)^2 (M_0 - \langle M_0 \rangle) \rangle,$$

$$r_{2a} = 2c_2 \sum_{n=0}^{N-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot (\mathbf{R}_n - \mathbf{R}_0)} \langle (M_n - \langle M_n \rangle)^2 (M_0^2 - \langle M_0^2 \rangle) \rangle, \\ r_{2b} = 2c_2 \sum_{n=0}^{N-1} e^{i(\mathbf{k}' - \mathbf{k}) \cdot (\mathbf{R}_n - \mathbf{R}_0)} \langle (M_n - \langle M_n \rangle) (M_n^2 - \langle M_n^2 \rangle) \rangle \\ \times (M_0 - \langle M_0 \rangle).$$

These generalize the on-site $n = 0$ terms, which we denote as r_1^o or r_2^o , of Kondo. There are now two distinct contributions at fourth order in spin, r_{2a} and r_{2b} . All off-site coefficients ($n \neq 0$) depend on the momentum transfer, making integrals slightly more complex than Kondo's.

Inverse spin Hall effect. Here there are two spin polarizations possible: first, that of the spin current, introduced by a spin injector, assumed to have a much higher Curie temperature than the T_C of the ferromagnetic metal (spin detector) in which the skew scattering and the inverse spin Hall effect occur. It is the injector which defines the direction of quantization of the spin current $\boldsymbol{\zeta}$ both above and below T_C . The second is the ordered moment of the ferromagnetic metal (spin detector), which we assume to be parallel to the spin injector below T_C . The incident spin currents are driven by the spin diffusion field: the difference in gradients of two electrochemical potentials ε_F^\pm [9, 10], $\mathcal{F} = -\frac{1}{e} \nabla(\varepsilon_F \pm \delta\varepsilon_F)$ with magnitude \mathcal{F} and direction $\pm\boldsymbol{\sigma}$ for the spin states \pm . The spin current is scattered to create a transverse voltage. Solving the Boltzmann equations for distributions over the two Fermi surfaces of spin-up and spin-down electrons using the skew scattering probabilities Eq. (3), the current densities [8] can be calculated as

$$\mathbf{j}^\pm = (\sigma_\parallel/2) \mathcal{F} \left\{ \pm(\boldsymbol{\sigma}) \pm (\Psi_\pm/\Phi)(\boldsymbol{\zeta} \times \boldsymbol{\sigma}) \right\},$$

where

$$\frac{\Psi_\pm}{\Phi} = (\Lambda_1 F_2 V / 3\pi N) (2m^3)^{1/2} \hbar^{-3} E_k^{1/2} \frac{B_\pm}{A}, \quad (4)$$

$$B_\pm = \frac{1}{8\pi} \int_0^\pi \int_0^{2\pi} \{ (r_1 \pm r_{2a}) (3F_0^2 + 4F_1^2 \cos \theta) \\ + (r_1 \pm r_{2b}) (-4F_0 F_1 - 8F_1^2 \cos \theta) \} \sin^3 \theta d\theta d\phi.$$

and

$$A = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \{ |J(\mathbf{k}, \mathbf{k}')|^2 p_1(\mathbf{k}' - \mathbf{k}) \\ + P(\mathbf{k}, \mathbf{k}') \} (1 - \cos \theta) \sin \theta d\theta d\phi.$$

p_1 depends on all components of momentum transfer as it includes spin-spin correlations between different ions. θ and ϕ are the angles for spherical coordinates with respect to the longitudinal current. σ_\parallel^{-1} is the resistivity given in terms of the integral A ,

$$\sigma_\parallel^{-1} = \rho_\parallel = (3\pi m / 2\hbar e^2) (V/N) (A/E_F). \quad (5)$$

Thus, the spin current $\mathbf{j}^s \equiv \mathbf{j}^+ - \mathbf{j}^-$ and the charge current $\mathbf{j}^c \equiv \mathbf{j}^+ + \mathbf{j}^-$ are

$$\mathbf{j}^s = \sigma_{\parallel} \mathcal{F} \boldsymbol{\sigma} + \sigma_{\parallel} \mathcal{F} \{(\Psi_+ + \Psi_-)/(2\Phi)\} (\boldsymbol{\zeta} \times \boldsymbol{\sigma}), \quad (6)$$

$$\mathbf{j}^c = \sigma_{\parallel} \mathcal{F} \{(\Psi_+ - \Psi_-)/(2\Phi)\} (\boldsymbol{\zeta} \times \boldsymbol{\sigma}). \quad (7)$$

As schematically shown in Fig. 1(a), for the incident spin current in the ISHE configuration, the spin-up (+) and spin-down (-) conduction electrons are scattered by skew scattering to the same side with the transition probability proportional to the terms $r_1 + r_2$ (r_2 represents r_{2a} and r_{2b}) and $-r_1 + r_2$, respectively. As a result, the Hall current \mathbf{j}^c is proportional to r_2 .

The Hall resistivity is defined as $\rho_{\text{ISH}} = E_{\perp}/j_{\parallel}$. It has $j_{\parallel} = \sigma_{\parallel} \mathcal{F}$ from Eq. (6), and $E_{\perp} = \mathcal{F}(\Psi_+ - \Psi_-)/(2\Phi)$ from Eq. (7). In the direction $\boldsymbol{\zeta} \times \boldsymbol{\sigma}$, the spin current does not contribute to the electric field E_{\perp} . Combining Eqs. (4) and (5), the inverse spin Hall resistivity ρ_{ISH} can be obtained as

$$\rho_{\text{ISH}} = \Lambda_1 B_2 F_2 (V/N)^2 (2m^5)^{1/2} / (2\hbar^4 e^2 E_F^{1/2}), \quad (8)$$

where $B_2 = (B_+ - B_-)/2$. B_2 involves only r_{2a} and r_{2b} :

$$B_2 = \frac{1}{8\pi} \int_0^{\pi} \int_0^{2\pi} \{r_{2a}(3F_0^2 + 4F_1^2 \cos \theta) + r_{2b}(-4F_0F_1 - 8F_1^2 \cos \theta)\} \sin^3 \theta d\theta d\phi.$$

We find that the temperature variation of the Hall resistivity ρ_{ISH} for the inverse spin Hall effect in a ferromagnetic metal is that of B_2 , *i.e.*, fourth-order correlations. The other factors in Eq. (8) do not show a significant temperature dependence close to the magnetic phase transition [8]. Note the on-site term $r_{2a}^o = r_{2b}^o$ is independent of angles θ and ϕ so the integral gives, in a purely local approximation, $\rho_{\text{ISH}}^o \propto \Lambda_1 r_2^o (F_0^2 - 4F_0F_1/3)F_2$. This is analogous to the local approximation ρ_{xy} for the anomalous Hall resistance obtained by Kondo, but appears to compare less well with experiment [7]: For the ISHE the non-local terms are more significant.

Spin Hall effect. For this effect [9] a transverse spin current is generated from a charge current, so we can deduce the current equations from those of the inverse effect by interchanging the spin and charge labels of the currents and replacing the spin diffusion field \mathcal{F} by the driving external electric field \mathcal{E} . To define a spin Hall transport coefficient we must specify the geometry of the measurement. We consider the non-local device scheme [9] where the potentials are measured for parallel (V_{\perp}^P) and antiparallel (V_{\perp}^{AP}) magnetizations of the injecting and detecting ferromagnetic electrodes. Both V_{\perp}^P and V_{\perp}^{AP} are proportional to $\mathcal{E}(\Psi_+ - \Psi_-)/(2\Phi)$ with different coefficients of proportionality. Ψ_+ and Ψ_- follow from Eqs. (4) and (5) as before, so the spin accumulation signal $\rho_{\text{SH}} = (V_{\perp}^P - V_{\perp}^{AP})/j_{\parallel} \propto \rho_{\text{ISH}}$. The proportionality constant depends on the transparencies of the barriers, which should be essentially independent of temperature, so that the temperature variation of the spin Hall resistivity ρ_{SH} is just that of the inverse spin Hall coefficient ρ_{SH} , *i.e.*, B_2 .

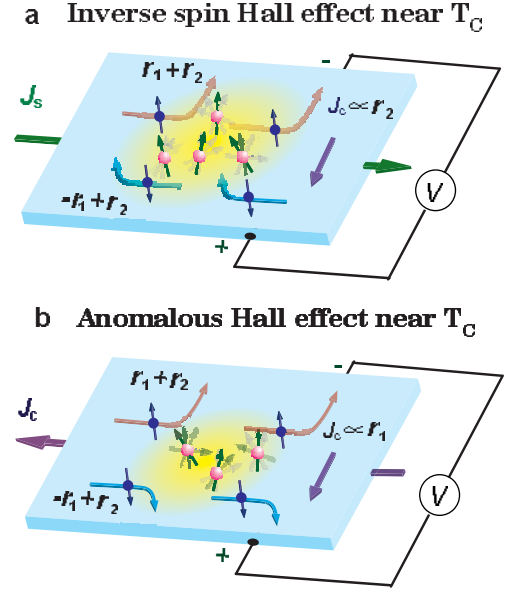


FIG. 1. (Color online) Schematic picture of (a) ISHE and (b) AHE near T_C . The skew scattering amplitudes for spin-up and spin-down conduction electrons (represented by arrows on the deflecting arrows) are proportional to $r_1 + r_2$ and $-r_1 + r_2$, respectively. The potentials measured depend on correlations of 4 (respectively 3), local spins in (a) [respectively (b)].

Anomalous Hall effect. We also generalized the expression for the AHE to include short-range spin-spin correlations. By interchanging the spin and charge labels of the currents and replacing the spin diffusion field \mathcal{F} by the external electric field $\mathcal{E} = -\nabla\phi$ with magnitude \mathcal{E} and direction $\boldsymbol{\sigma}$ for the spin states \pm , the current densities [8] can be calculated as

$$\mathbf{j}^{\pm} = (\sigma_{\parallel}/2) \mathcal{E} [\boldsymbol{\sigma} + (\Psi_{\pm}/\Phi) \{\boldsymbol{\zeta} \times \boldsymbol{\sigma}\}].$$

Thus, the spin and charge currents are obtained as

$$\mathbf{j}^s = \sigma_{\parallel} \mathcal{E} \{(\Psi_+ - \Psi_-)/(2\Phi)\} (\boldsymbol{\zeta} \times \boldsymbol{\sigma}), \quad (9)$$

$$\mathbf{j}^c = \sigma_{\parallel} \mathcal{E} \boldsymbol{\sigma} + \sigma_{\parallel} \mathcal{E} \{(\Psi_+ + \Psi_-)/(2\Phi)\} (\boldsymbol{\zeta} \times \boldsymbol{\sigma}). \quad (10)$$

As shown in Fig. 1(b), for the incident charge current in the AHE configuration, the spin-up (+) and spin-down (-) electrons are scattered by skew scattering to opposite sides with the transition probability proportional to the terms $r_1 + r_2$ and $-r_1 + r_2$, respectively. As a result, the Hall current part in \mathbf{j}^c is proportional to r_1 .

The Hall resistivity is defined as $\rho_H = E_{\perp}/j_{\parallel}$. Now $j_{\parallel} = \sigma_{\parallel} \mathcal{F}$ from Eq. (9), and $E_{\perp} = \mathcal{F}(\Psi_+ - \Psi_-)/(2\Phi)$ from Eq. (10). In the direction $\boldsymbol{\zeta} \times \boldsymbol{\sigma}$, the spin current does not contribute to the electric field E_{\perp} . With Eqs. (4) and (5), the anomalous Hall resistivity ρ_H can be obtained as

$$\rho_H = \Lambda_1 (V/N)^2 (2m^5)^{1/2} B_1 F_2 / (2\hbar^4 e^2 E_F^{1/2}),$$

where $B_1 = (B_+ + B_-)/2$. B_1 involves only r_1 :

$$B_1 = \frac{1}{8\pi} \int_0^\pi \int_0^{2\pi} r_1 \{3F_0^2 - 4F_0F_1 - 4F_1^2 \cos \theta\} \sin^3 \theta d\theta d\phi.$$

If we consider only the site-diagonal $n = 0$ term in $r_1 = r_1^o$, $B_1 = r_1^o(F_0^2 - 4F_0F_1/3)$, and ρ_H will reduce to Kondo's result. We remark that the definitions of the local r_1^o and r_2^o terms were given by Kondo. Only the r_1^o term was found to contribute to the AHE [8]. The effect of the r_2^o term, on the other hand, has been hidden for about 50 years. Of course Kondo's paper was written almost a decade before Dyakonov and Perel's [1], whose ideas have been explored even more recently with experimental manipulation of spin currents.

The difference between the AHE and the ISHE near T_C originates from the different symmetries of the incident charge current in the AHE and incident spin current in the ISHE, as in Fig. 1. This symmetry difference gives rise to the distinct orders of spin-spin correlation: Near T_C the temperature variation of ρ_H in the AHE is determined by a three-spin correlation r_1 , while the ρ_{SH} in the ISHE is determined by four-spin correlations r_{2a} and r_{2b} . The difference in order can be traced back to the different terms linear in spin orbit in Eq. (1): For charge current the term linear in local spin S_n combines with two exchange terms (for the second-order Born approximation there are three powers of the Hamiltonian) to give a third-order term in local spins, whereas for spin current any of the last three terms, with *two* local spin operators, combine with two exchange terms to give a four-spin operator. In particular, the third-order correlations vanish by symmetry above T_C whereas the fourth order does not. In the limit of zero momentum transfer, i.e., $\mathbf{k} - \mathbf{k}' = 0$, r_1 and r_{2a} ($=r_{2b}$) simplify into first- (χ_1) and second-order (χ_2) non-linear uniform susceptibilities [11], respectively, as shown in Fig. 2. The divergence at T_C should be smeared out in the r_1 and r_{2a} and r_{2b} due to the finite momentum transfer $\mathbf{k} - \mathbf{k}'$, as for the resistance [12]. As a result, the different symmetries should be reflected by a single peak below T_C in the ρ_H and two peaks of opposite sign above and below T_C in the ρ_{SH} .

Discussion. This theory is formulated for a pure crystalline metal and it would normally be considered as an “intrinsic” mechanism. In fact, the dependence of the Hall constant on longitudinal resistance resembles more what is classified as “extrinsic” models for skew scattering from impurities. This is not an accident, since it is the fluctuations of the local moments of d orbitals which skew scatter close to the Curie temperature. Fert and co-workers [13], in a study of dilute alloys which are truly “extrinsic” in the sense of including impurities, approached the transport in a way that looks different, but is actually compatible. Rather than finding scattering amplitudes from microscopic potentials, they replaced the d orbitals by a set of phase shifts to be determined empirically. They included p - and d -wave phase shifts, more significant in the alloys, rather than

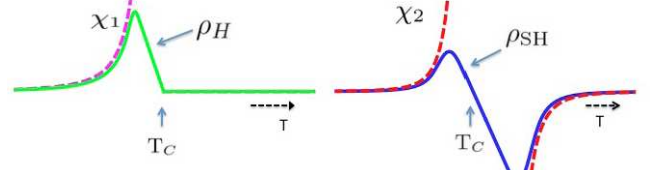


FIG. 2. (Color online) Schematic behavior of the anomalous (left) and spin Hall resistances (right) around T_C . In each case the power-law divergence (dashed lines) of the corresponding non-linear susceptibility χ_1 or χ_2 is cut off by the finite Fermi surface. The different shapes (solid lines) of the anomalies reflect that the third-order correlations vanish about T_C and the fourth-order correlations change sign across T_C .

the s - and p -scattering channels included here. As it is enough for skew scattering to include interference from phase shifts of different parities, the real difference is in the inclusion of resonant effects. We remark also that expression $\rho_H \propto r_1^o(F_0^2 - 4F_0F_1/3)\Lambda_1F_2$, obtained in the site diagonal limit is the equivalent of that of Fert *et al.*: $\rho_H \propto \lambda_d \sin \delta_p \sin(2\delta_d - \delta_p) \sin^2 \delta_d$ with Λ_1F_2 (coming from the scattering amplitude of the p -wave exchange amplitude and the local spin via the spin-orbit coupling) corresponding to the change in phase shifts for different components of d -wave scattering from the resonant potential $\lambda_d \sin^2 \delta_d$. The factor $(F_0^2 - 4F_0F_1/3)$ corresponds to the term $\sin \delta_p \sin(2\delta_d - \delta_p)$, while the former term representing interference between the s wave (F_0) and p wave (F_1), and the latter term representing interference between the p wave and d wave. The similarity ends here: There is no equivalent of r_1^o in Fert's theory, which was formulated for alloys at low temperatures, while here we are, as in Ref. [8], interested in the collective spin fluctuations near criticality. The distinction between the anomalous and spin Hall resistivities depends on the difference between the r_1 and the two r_2 terms. We remark that many-body effects can be included by use of the calculated temperature-dependent phase shifts for an isolated quantum impurity [4] or in lattice models [14] for mixed-valence systems where there is a temperature-dependent effective phase shift.

Our theory includes skew scattering but neither side-jump nor the “topological” lattice effects coming from anomalous velocities generated by spin-orbit terms [15, 16] and should thus be applicable when the conductivity is large [17]. The band theories can be extended to finite temperatures by replacing the magnetization of the band by the temperature-dependent value $\rho_H \propto \rho_{||}^2(M)$ but it is hard to see how the anomalous velocity could produce non-monotonic behavior *above* T_C , in contrast to our approach.

Comparison with experiments. Recently Wei *et al.* [7] have performed an inverse spin Hall effect experiment in a weak ferromagnetic alloy near T_C , where a dip and a peak are observed in the spin Hall resistivity ρ_{SH} be-

low and above the T_C , respectively. We argue that our theory can qualitatively explain the experimentally observed anomalous behaviors in ρ_{SH} near T_C . The theory we provide here including correlations between spins on more than one atom seems to be necessary to explain the experiments. From simulations [7] on short-range Heisenberg models the on-site terms are rather smooth and only the off-site terms are non-monotonic. Thus features close to T_C may be dominated by the correlations that are off-diagonal in the site index.

Conclusions. We have presented a theory of the inverse spin and spin Hall effects that provides an essential complement to the classic theory of Kondo for the anomalous Hall effect, which has remained unchallenged for 50 years. We have found an essential difference between the anomalous Hall effect and the inverse spin Hall effect near T_C in a ferromagnetic metal. Our theory can be compared to recent experimental results for the SHE

in ferromagnetic alloys.

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